

# Stabilizing the axion and a natural solution to the $\mu$ problem of supersymmetry\*

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The axion solution to the strong CP problem makes use of a global Peccei-Quinn (PQ)  $U(1)$  symmetry which is susceptible to violations from quantum gravitational effects. We show explicitly how discrete gauge symmetries can protect the axion from such violations. PQ symmetry emerges as an accidental global symmetry from discrete gauge symmetries which are subgroups of the anomalous  $U(1)$  of string origin. We also show how the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion model provides a natural solution to  $\mu$  problem of supersymmetry as  $\mu \sim M_{\text{SUSY}} \sim M_{\text{PQ}}^2/M_{\text{Pl}}$ .

## I. THE STRONG CP PROBLEM AND STABILIZING THE AXION

CP violation (CPV) can exist in the QCD Lagrangian arising from the instanton induced Chern-Simons type gluon-gluon coupling

$$\mathcal{L} \supset \theta g_s^2 \epsilon^{\mu\nu\sigma\rho} G_{\mu\nu}^\alpha G_{\sigma\rho}^\alpha / 64\pi^2 = \theta g_s^2 G_{\mu\nu}^\alpha \tilde{G}^{\alpha\mu\nu} / 32\pi^2.$$

In addition, there is another CPV source from the quark mass matrices. This results in an observable parameter  $\bar{\theta}$  defined as

$$\bar{\theta} = \theta + \arg(\det M_U \det M_D).$$

Such a  $\bar{\theta}$  would lead to a neutron electric dipole moment (EDM) of order  $d_n \simeq 5 \times 10^{-16} \bar{\theta}$  ecm, while the current experiment limit is  $d_n < 10^{-25}$  ecm. This puts a strong constraint,  $\bar{\theta} < 10^{-10}$ . Peccei-Quinn (PQ) symmetry[1] is an elegant solution to this so-called strong CP problem. It introduces a global  $U(1)$  symmetry, broken by the QCD anomaly, which generates a pseudo-Goldstone particle  $a$ , the axion. Non-perturbative effect then induces a term in the Lagrangian

$$\mathcal{L} \supset (a/f_a) g_s^2 G_{\mu\nu}^\alpha \tilde{G}^{\alpha\mu\nu} / 32\pi^2.$$

$\bar{\theta}$  is then promoted to this dynamical field axion as  $a(x)/f_a$ . Minimizing the axion potential

$$V(a) \propto \Lambda_{\text{QCD}}^4 (1 - \cos(a(x)/f_a)),$$

consequently  $\bar{\theta} = \langle a \rangle / f_a = 0$ . [11] The strong CP problem is then solved.  $f_a$  is the (model dependent) axion decay constant[2] and it is constrained to be  $f_a = (10^{10} - 10^{12})$  GeV by the combined limits from laboratory experiments, astrophysics and cosmology. Hence, only the “invisible axion” models, which have appropriate values of  $f_a$ , are favored.[3, 4] The couplings of the axion with the Standard Model (SM) fields are highly suppressed in these models. Although axion arise as a pseudo-Goldstone particle when the PQ symmetry is explicitly broken by its QCD anomaly, the axion can acquire a tiny mass through higher order non-perturbative effect. The mass of the axions can be estimated to be

$$m_a \sim \Lambda_{\text{QCD}}^2 / f_a \sim 10^{-4} \text{ eV}.$$

Quantum gravitational effects can potentially violate the global PQ symmetry as they can break all global symmetries while respecting gauge symmetries. In the axion models, a possible quantum gravity generated non-renormalizable term

$$\mathcal{L} \supset S^n / M_{\text{Pl}}^{n-4}$$

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is in principle allowed. This term would lead to

$$\bar{\theta} \simeq f_a^n / (M_{\text{Pl}}^{n-4} \Lambda_{\text{QCD}}^4).$$

Since both  $\bar{\theta}$  and  $f_a$  are highly constrained,  $n \geq 10$  is necessary. To avoid such kind of violations, one solution is to introduce a discrete gauge symmetry. The PQ symmetry arises only as an accidental global symmetry from it. Discrete gauge symmetries[5] are remnants of gauge groups after spontaneous symmetry breaking and they are left intact by quantum gravity. There is no gauge boson associated with such a symmetry. However, for it to be of gauge origin, it must be anomaly free.

Conventionally, absence of anomalies complicates the particle spectrum of axion models. However, the Type I and Type IIB string theories provide a new candidate that cancels the anomalies without enlarging the particle content. In the low energy effective theory of such string theories, there exists one anomalous  $U(1)_A$  symmetry. The Green-Schwarz Mechanism (GSM)[6] is effective in cancelling the anomalies. The anomalous  $U(1)_A$  symmetry is broken by a Higgs field spontaneously near the string scale. If this Higgs has charge  $N$  under  $U(1)_A$ , there will be a remnant  $Z_N$  discrete gauge symmetry. At low energy, one can check if  $Z_N$  can be embedded in to the anomalous  $U(1)_A$  by a discrete version of GSM. It requires

$$(A_3 + \text{mod } N/2)/k_3 = (A_2 + \text{mod } N/2)/k_2 = (A_1 + \text{mod } N/2)/k_1,$$

where  $A_i$  are mixed anomaly coefficients with the SM gauge group  $G_i$  as  $A_{[G_i]^2 \times Z_N}$  and  $k_i$  are the corresponding Kac-Moody levels. Notice under a  $Z_N$  symmetry, the anomaly coefficient can differ by mod  $N/2$  since one can always have vectorial particles in the theory which do not contribute to the anomalies at the  $U(1)$  level.

Here we take the supersymmetric (SUSY) DFSZ axion model as an explicit example. The superpotential of the DFSZ axion model contains a term  $\lambda H_u H_d S^2 / M_{\text{Pl}}$ . After  $H_u$ ,  $H_d$  and  $S$  develop VEVs, the global PQ symmetry is broken and the axion arises as a pseudo-Goldstone particle. Since the superpotential is holomorphic, one cannot write  $S^{\dagger 2} S^2$  type term. In addition to the  $S$  field, another singlet  $\tilde{S}$  is needed so that the axion is invisible and at the same time, PQ can be broken. The superpotential of the model now is

$$W \supset \lambda_1 H_u H_d S^2 / M_{\text{Pl}} + \lambda_2 S^2 \tilde{S}^2 / M_{\text{Pl}}.$$

One explicit example of  $Z_{22}$  discrete gauge symmetry is given. The charge assignment under  $Z_{22}$  is listed as

$$\{Q = 3, u^c = 19, d^c = 1, L = 11, e^c = 15, \nu^c = 11, H_u = 22, H_d = 18, S = 13, \tilde{S} = 20\}.$$

The mixed anomalies are  $\{A_2 = 6, A_3 = 17\}$ . It apparently satisfies the GSM condition.  $S^{22}/M_{\text{Pl}}^{19}$  is the leading allowed term in the superpotential due to potential quantum gravity correction, which only induces  $\theta \lesssim 10^{-130}$ .

In this model, the  $R$ -parity is not automatic, for instance,  $L H_u S \tilde{S}$  is allowed. To get an exact  $R$ -parity, one can introduce an additional  $Z_2$  where all the SM matter fields are odd but  $H_u$ ,  $H_d$ ,  $S$  and  $\tilde{S}$  are even. This is the unbroken subgroup of the gauge symmetry  $U(1)_{\text{B-L}}$  even with the presence of Majorana neutrino mass term.

The KSVZ Axion model[4], can also be stabilized by discrete gauge symmetries.[7] It requires extra matter fields which are vectorial under QCD. An interesting case occurs when the matter fields include two fields and together form a fundamental representation of the  $SU(5)$  GUT group. The theory is then automatically consistent with GSM, while providing nonzero QCD anomaly.

## II. DFSZ AXION: A NATURAL SOLUTION TO THE $\mu$ PROBLEM

The  $\mu$  problem is an intriguing puzzle of SUSY extension of the SM where  $\mu$  is the Higgs mass parameter in

$$W \supset \mu H_u H_d$$

. Why should  $\mu$  be of order the soft SUSY breaking mass scale  $M_{\text{SUSY}}$ , rather than the Planck scale  $M_{\text{Pl}}$ . There are two well-known attempts to solve this problem. One is through a non-renormalizable Kähler potential,

$$\mathcal{L} \supset \int d^4\theta H_u H_d Z^* / M_{\text{Pl}},$$

namely the Giudice-Masiero mechanism with an additional  $R$ -symmetry. Another one is the NMSSM models with a global  $Z_3$  symmetry at low energy or a  $U(1)'$  gauge symmetry broken near  $M_{\text{SUSY}}$ .

Imposing a new physics scale  $M_{\text{PQ}}$  ( $f_a = (10^{10} - 10^{12})$  GeV), the axion models provide another approach to the  $\mu$  problem[7, 8] as

$$\mu \sim M_{\text{PQ}}^2/M_{\text{Pl}}.$$

In the case of DFSZ axion model, a  $\mu$  term automatically arises after PQ symmetry breaks.

The question now is how to naturally understand the origin of  $M_{\text{PQ}}$  from a higher energy theory. It is interesting that in the SUGRA mediated SUSY breaking models, one also has to impose a new physics scale of order  $\mathcal{O}(10^{11})$  GeV. In these models, this intermediate scale can be generated dynamically. Practically, this intermediate scale can then be identified as  $M_{\text{PQ}}$ . Here we propose a model involving SUSY breaking.[9] Having made use of  $M_{\text{SUSY}}$ , this approach certainly requires that SUSY breaking mediation scale is greater than  $M_{\text{PQ}}$ . A simple realization of this idea is the SUGRA model. The superpotential of the model contains

$$W \supset \lambda_1 H_u H_d S^2/M_{\text{Pl}} + \lambda_2 (S\tilde{S})^2/M_{\text{Pl}} + S^{22}/M_{\text{Pl}}^{19}$$

which is also consistent with the  $Z_{22}$  symmetry in the previous section. By minimizing the leading-orders potential including SUSY breaking effects,

$$V = (\lambda_2 C (S\tilde{S})^2/M_{\text{Pl}} + h.c) + m_S^2 |S|^2 + m_{\tilde{S}}^2 |\tilde{S}|^2 + 4\lambda_2 |S\tilde{S}|^2 (|S|^2 + |\tilde{S}|^2)/M_{\text{Pl}}^2,$$

where  $m_S$  and  $m_{\tilde{S}}$  are soft breaking masses of order  $M_{\text{SUSY}}$ , one obtains

$$f_a^2 = C \pm \sqrt{C^2 - 12m_S^2 M_{\text{Pl}}}/12\lambda_2.$$

So

$$f_a \sim \sqrt{M_{\text{Pl}} M_{\text{SUSY}}} \sim 10^{11} \text{ GeV}.$$

Since the  $F$ -component of the field  $S$ ,

$$F_S \sim M_{\text{PQ}} M_{\text{SUSY}},$$

the dominant contribution for the  $B$  parameter which appears in the soft bilinear SUSY breaking term

$$\mathcal{L}_{\text{soft}} \supset B\mu H_u H_d$$

arises from the superpotential  $H_u H_d S^2/M_{\text{Pl}}$  as

$$B\mu = \langle S \rangle \langle F_S \rangle / M_{\text{Pl}} \sim M_{\text{SUSY}}^2.$$

So it is difficult to distinguish it from the usual MSSM via electroweak physics. However, as the axion can be a cold dark matter candidate, one can still distinguish the model in cosmology. In this model, the two PQ Higgs bosons have masses of order  $M_{\text{SUSY}}$  but their mixings with the doublet Higgs are highly suppressed. The orthogonal combination to the axion acquires a mass of order  $M_{\text{SUSY}}$ . The axino and saxino masses are both around  $M_{\text{SUSY}}$ . The axino can mix with the Higgsino with a tiny mixing angle of order  $(M_{\text{SUSY}}/M_{\text{Pl}})^{1/2} \sim 10^{-7}$ . Therefore, the axino can decay to a bottom quark and a sbottom squark with a lifetime

$$\tau \sim 10^{-11} \text{ sec}.$$

This is a consistent picture with big-bang cosmology since the axino decays occur earlier than the nucleosynthesis era.

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